EFFECT OF ASTRONOMICAL FACTORS ON ENERGY DENSITY VARIATIONS IN THE EARTH SOLID MANTLE*

Boris W. Levin[1], Vladimir P. Pavlov[2]

[1] P. P. Shirshov Institute of Oceanology RAS, Moscow, Russia
[2] Steklov Institute of Mathematics RAS, Moscow, Russia
E-mail: **levin@rfbr.ru**

The idea of possible linkage between tectonic processes in the lithosphere and astronomical factors, such as tidal forcing, irregularity of Earth rotation and Chandler wobble (pole displacement), has been repeatedly discussed and recently analyzed in details in the geophysical literature [Wahr, 1985; Gor'kavy, 1989; Chao, 1995; Avsjuk, 1996; Levin, 1996]. A more universal approach, which allows to estimate the influence of these factors on the state of the stressed lithosphere, is proposed here. We base this approach on the following modern model of the structure of the Earth: the Earth is a spheroid consisting of a solid crust and a viscous liquid filling its spherical interior, with a denser solid core inside.

The nature of tectonic motions and driving forces, which move plates and slabs of the Earth crust, is still a subject of considerable discussion. Traditionally, crust motion and the forces that cause it are associated with various processes deep inside the Earth, such as convective currents, gravitational and chemical differentiation and effects of plumes [Khain, 1973; Zonenstein, 1993; Pushcharovsky, 1999].

It was also postulated that external astronomical factors may have a significant effect on the tectonic and seismic processes [Kant, 1756; Darwin, 1879; Mayer, 1893; Khain, 1960; Kropotkin, 1963; Nalivkin, 1963]. Recently, considerable attention was focused on the influence of the Earth core motion on the various geophysical processes [Avsjuk, 1973; Jacobs, 1995; Avsjuk, 1998], on the consequences of the Earth center displacement [Avsjuk, 1999], and on the analysis of time correlation between regional seismicity and irregular rotation of the Earth [Gor'kavy, 1999]. Levin [1999] has analyzed the specific features of meridional distribution of Earth seismicity and has provided well-grounded arguments confirming the correlation between the seismicity and the Earth rotation.

According to Avsjuk [1973, 1996], the solid core moves inside the liquid core subject to gravitational forces of Moon and Sun. Displacements of the solid core with an amplitude of the order of 100 m and specific periods that are known from astronomical observations (approximately 14, 365, 412-437 days and 6-7 years) produce displacements of the Earth center of gravity with the same periods and an amplitude of about 4 m. The Earth rotation axis follows the position of the gravity center and moves inside the Earth body. Consequently, the poles move as well, producing the Chandler wobble of the poles [Chandler, 1892]. These tidal forces also influence the tectonics of the lithosphere [Nadai, 1969; Sadovsky *et al*.,1987].

Model. We propose a model, which combines the effects of irregular Earth rotation, Chandler wobble and tidal forces on the stressed state of the lithosphere. Density variations in the free litospheric energy provide a qualitative example of such effects. These variations can be calculated using the linear theory of elasticity. They are represented by a stress tensor, which can be reconstructed from equilibrium conditions of a lithosphere element subjected to all of the forces.

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It is convenient to write the equilibrium conditions in a rotating coordinate system. In this case, the revolution axis passes through the center of gravity of the Earth, and the lithosphere element is at rest. Let r be the radius vector of the lithosphere element (from the origin, or the Earth gravity center), ρ the density of an element, with all quantities defined per unit volume. The net force on a motionless element is zero:

$$(1) \qquad \mathbf{0} = -\rho \frac{\partial \varphi}{\partial \mathbf{r}} - \rho\,[\Omega[\Omega \mathbf{r}]] + \rho\,\ddot{\mathbf{x}} + \rho\,(\gamma M_* |\mathbf{R}_* - \mathbf{r}|^{-3} (\mathbf{R}_* - \mathbf{r}) - \mathbf{a}_*) + \mathbf{K}\,.$$

Here the first term on the right side is due to the Earth gravity, the second term is the centrifugal force (Ω is the angular velocity vector of the Earth rotation), the third term is inertia force due to irregularity of the displacement $\mathbf{x}(t)$ of the Earth center of gravitation, the forth term is due to the tidal forcing from any astronomical body with mass M_*, separated from Earth by a distance R_*, (γ is the Newtonian gravitational constant, a_* is a relative acceleration between the Earth and this body),.The Coriolis force is assumed to be zero. The last term is the force produced by the stress in the crust and is the divergence of the stress tensor (where p is the pressure):

$$(2) \qquad K_i = \partial \sigma_{ik} / \partial r,\ \sigma_{ik} = -\delta_{ik} p + r^{-2}(r_i r_k - 3^{-1}\delta_{ik} r^2)s\,.$$

All forces, except those caused by the stress, are the gradients of the corresponding potentials (multiplied by the density). These potentials are: gravity φ_{Gr}, centrifugal $\varphi_{\mathrm{Cent}} = 2^{-1}(\Omega^2 \mathbf{r}^2 - (\Omega \mathbf{r})^2)$, the potential of inertia forces $\varphi_{\mathrm{Iner}} = \mathbf{r}\ddot{\mathbf{x}}$, and the tidal potential

$$\varphi_{\mathrm{Tide}} = \gamma M_* |\mathbf{R}_* - \mathbf{r}|^{-1} - \mathbf{r}\mathbf{a}_* \approx 2^{-1}\gamma M_* R_*^{-5}(r^2 R_*^2 - 3(\mathbf{r}\mathbf{R}_*)^2)$$

To make the model more realistic, we can approximate the gravity potential as a homogeneous ellipsoid of revolution

$$(3) \qquad \varphi_{\mathrm{Gr}} = \varphi_0 - 2\pi\gamma\,\rho_0 R^2 (1 - (\alpha + \beta \cos^2\theta)(\mathbf{r}/R)^2),$$

where the parameters φ_0, α and β are chosen from the condition that the sum of gravity and centrifugal potentials includes the surface of the rotation ellipsoid as a level surface (in this case the Earth surface will be described in the same way). Here ρ_0 is mean density of the Earth, R is polar radius, θ is a latitude of the lithosphere element.

Evaluation of the potentials. Let us evaluate the order of magnitudes of the potentials. Taking $\gamma = 6.7 \cdot 10^{-8}\,\mathrm{cm^3 g^{-1} s^{-2}}$, $\rho_0 \approx 5\ \mathrm{g\,cm^{-3}}$, and $R \approx 6.4 \cdot 10^8\,\mathrm{cm}$, we obtain $\varphi_{Gr} \approx 8.6 \cdot 10^{11}\,\mathrm{cm^2 s^{-2}}$. For the centrifugal potential, if $|\Omega| \approx 7.3 \cdot 10^{-5}\,\mathrm{s^{-1}}$, then $\varphi_{\mathrm{Cent}} \approx 2^{-1}\Omega^2 R^2 \approx 1.1 \cdot 10^9\,\mathrm{cm^2 s^{-2}}$. We showed previously [Levin and Pavlov, 2001], that the magnitude of the potential for inertia force is given by $Ry\omega^2 \cdot 10^{-1} \approx 2.6 \cdot 10^{-3}\,\mathrm{cm^2 s^{-2}}$, where $\omega \approx 2 \cdot 10^{-7}\,\mathrm{s^{-1}}$ is the angular velocity of the Earth revolving around the Sun, $y \approx 10^3\,\mathrm{cm}$ is the amplitude of the Chandler wobble. The Moon tidal potential is expressed in the form: $\varphi_{\mathrm{Tide}} \approx 2^{-1}\gamma M_* R^2 R_*^{-3} \approx 1.8 \cdot 10^4\,\mathrm{cm^2 s^{-2}}$. Finally, if we take into the account the irregularity of the Earth rotation (i.e. forced precession of 50'' per year corresponding to the contribution of the vector with length $\varepsilon \approx 1.2 \cdot 10^{-12}\,\mathrm{s^{-1}}$, normal to the ecliptic plane, to the angular velocity Ω) then the centrifugal potential changes by: $\varphi_{\mathrm{Prec}} = ((\Omega\varepsilon)r^2 - (\varepsilon\,\mathbf{r})(\Omega \mathbf{r})) \approx \Omega\varepsilon\,R^2 \approx 3.6 \cdot 10^1\,\mathrm{cm^2 s^{-2}}$. All other potentials have small effects in comparison with gravity and centrifugal potentials. These effects can be calculated using the framework of the perturbation theory.

Contribution to free energy. The expression (2) for the stress tensor is an assumption of the model. Its first term is related to the volume, and the second term to the shear stress. From linear theory of elasticity we can express the density of free energy of a lithosphere element as:

(4) $$F = (18K)^{-1}\sigma_{ll}^{2} + (4\mu)^{-1}(\sigma_{ik}^{2} - 3^{-1}\sigma_{ll}^{2}) = (2K)^{-1}p^2 + (4\mu)^{-1}(2/3)s^2 .$$

We can use the so-called hydrostatic approximation as a zero-th order approximation of the perturbation theory. In this case the stress tensor is reduced to the first volume term. The equilibrium conditions take into account only gravitational and centrifugal potentials but do not include astronomical factors. Applying a first order perturbation, we obtain a small contribution δp to the pressure, while the scalar function s is also a small quantity of the first order. Hence, first order contribution to the free energy due to astronomic factors can be written as:

(5) $$\delta F = K^{-1} p \delta p .$$

The density ρ and the pressure p are dynamic properties of the state of stress, and, generally speaking, depend on the vector variables $\mathbf{r}$, Ω, ω, $\mathbf{a}_*$, $\mathbf{x}(t)$, and so on. If these dynamic characteristics can be described by analytical functions, we can conclude that they depend only on scalar combinations of these vector variables. We have shown previously [Levin and Pavlov, 2001] that the solvability condition of the equilibrium system leads us to conclude that this dependence is specific: the dynamic characteristics are the functions of one scalar variable ξ, which is simply the sum of all the potentials we used in the model. Moreover, the pressure is the integral of the density over this variable:

(6) $$p = -\int_{\xi}^{\xi^0} \rho(\xi) d\xi ,$$

where ξ^0 is the value of the variable ξ on the Earth surface. In this case, expression (5) for the contribution of pressure to free energy density becomes:

(7) $$\delta F = K^{-1} p \rho \delta \xi ,$$

where $\delta \xi = \varphi_{\text{Iner}} + \varphi_{\text{Prec}} + \varphi_{\text{Tide}}$ is a sum of the contributions due to gravity and centrifugal potentials. These account for the astronomic factors to a first-order approximation of the perturbation theory.

Meridian dependence. Generally speaking, all three factors in the right part of expression (7) depend on latitude of the lithosphere element. However, the pressure and the density are calculated in the zeroth-order approximation of the perturbation theory, while their argument ξ accounts only for gravity and centrifugal potentials. In this case the term proportional to $\cos^2\theta$ is small, of an order of 1/300 (Earth flatness), and it is possible to neglect the meridional dependence of pressure and density.

We showed [Levin and Pavlov, 2001] that for a coarse approximation for the Chandler wobble,

(8) $$\mathbf{y}(t) = y(\cos \nu t - 1,\ \sin \nu t,\ 0) ,$$

where $\nu / 2\pi$ is the frequency of a mode with maximum amplitude (corresponding to the period of 425 days), the displacement of the center of gravity $\mathbf{x}(t)$ can be approximately described by a similar expression (we neglect here declination differences between the Sun and the Moon):

$$\mathbf{x}(t) = y(\sin\gamma)^{-1}(\cos\nu t - 1,\ \sin\nu t,\ \cos\gamma),\ \cos\gamma = \sin\delta\cos\omega t, \quad (9)$$

where $\pi / 2 - \delta$ is a tilt angle of the rotation axis to the ecliptic, γ is an angle between the rotation axis and direction to the Sun (Figure 1a shows two extreme axis positions), ω is the angular velocity of Earth rotation around the Sun. In this case, the variation $\varphi_{\text{Iner}} = \mathbf{r}\ddot{\mathbf{x}}$ during one half of the Chandler period has the form

$$\Delta\varphi_{\text{Iner}} = k_{\text{Iner}}(x) f_{\text{Iner}}(\theta),\ k_{\text{Iner}} = 10^{-1} R y \omega^2 x,\ x = r/R,\ f_{\text{Iner}}(\theta) = 1.36\cos\theta + 0.6\sin\theta\,. \quad (10)$$

This function depends on latitude and has its maximum at about 23°.

In case of a forced precession, it is natural to estimate variation of the corresponding potential φ_{Prec} for half of a day. Two positions of the lithosphere elements separated by half a day are shown in Figure 1b. We then have:

$$\begin{aligned} &\Delta\varphi_{\text{Prec}} = \Omega\varepsilon r^2 \cos\theta\,(\cos(\frac{\pi}{2} - \theta - \delta) - \cos(\frac{\pi}{2} - \theta + \delta)) = k_{\text{Prec}}(x) f_{\text{Prec}}(\theta), \\ &k_{\text{Prec}}(x) = \Omega\varepsilon r^2 x^2, \qquad f_{\text{Prec}}(\theta) = \sin 2\theta \sin\delta \end{aligned}, \quad (11)$$

where δ is an angle between the equatorial plane and the ecliptic.

Finally, the variation of the tidal potential φ_{Tide} for half a day can be expressed as (see Figure 1b):

$$\begin{aligned} &\Delta\varphi_{\text{Tide}} = 2^{-1} 3\gamma M_* r^2 R_*^{-3}(\cos^2(\theta - \delta) - \cos^2(\theta + \delta)) = k_{\text{Tide}}(x) f_{\text{Tide}}(\theta), \\ &k_{\text{Tide}}(x) = 2^{-1} 3\gamma M_* R^2 R_*^{-3} x^2, \quad f_{\text{Tide}}(\theta) = \sin 2\theta \sin 2\delta \end{aligned}. \quad (12)$$

In the last two cases the latitude dependence function has its maximum at 45°.

Integral variation of free energy. It is useful to evaluate total variation of free energy of the lithosphere, which is due to astronomical effects. In order to do this, we prescribe the dependence of density on depth and integrate (7) over the surface and the radius-vector *r* down to a certain depth *h*. Let assume that *h* = 100 km and approximate the density increase with depth by a linear function (the calculations show that correction due to non-linearity is negligible in the transition zones [Levin and Pavlov, 2001]),

$$\rho = \rho_R(1 + \kappa(1 - r/R_\theta)),\ \kappa \approx 4{,}5, \quad (13)$$

where ρ_R is the density on the surface, and $R_\theta = R(1 + \varsigma\cos^2\theta)$ is the radius of the point on the surface at a given latitude. Parameter ς, which can be expressed using the parameters of the gravity potential, is of the order of 1/300. Then, for zero-th order in small parameter ς,

$$\Delta F_\nu = \int_{R-h}^{R} dr \int dS \delta\, F = K^{-1} R \int_{1-hR^{-1}}^{1} dx x p(x) \rho(x) k_\nu(x) \int_{0}^{1} d\cos\theta\, f_\nu(\theta) \qquad (14)$$

After simple calculations and for the same periods of time as in the evaluation of the latitude dependence, we get for Chandler wobble $\Delta F_{\text{Iner}} \approx 0.65 \cdot 10^{24}$ erg, for forced precession $\Delta F_{\text{Prec}} \approx 1.2 \cdot 10^{25}$ erg, and for tidal effects $\Delta F_{\text{Tide}} \approx 2.8 \cdot 10^{29}$ erg.

Discussion. Strictly speaking, precession and Chandler effects are not correctly accounted for in the first approximation of the perturbation theory. Indeed, the tidal potential expansion is based on the Laplace approximation, which is a leading order expansion in a parameter $R / R_* \approx 1.7 \cdot 10^{-2}$. The precession and Chandler potentials are, respectively, 3 and 7 orders of magnitude smaller than the tidal potential. However, we found it useful to present the corresponding calculations for these two factors as well.

Latitude functions calculated from the model of free-energy density variations (Figure 2a) coincide quantitatively with the latitude function of power fluxes during earthquakes (Figure 2b). They have a minimum at the equator and maxima at mid latitudes. Further, a comparison of earthquake energy fluxes with total free energy variation shows that the total energy fluxes during earthquakes are of the order of $10^{25} - 10^{26}$ erg. On the other hand, according to a general theory on periodic processes in non-ideal continuum, there are vibrational energy dissipation due to viscosity and also contributions due to local defects. Existing estimates of the energy dissipation are very rough; they depend on the non-ideal nature of the model, and are in the range of $10^{-2} - 10^{-3}$. We observe that tidal variations can be one of the energy sources during earthquakes.

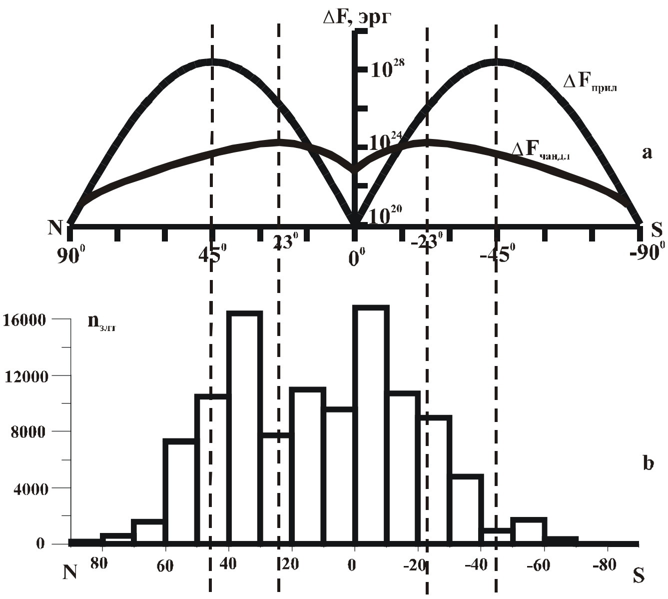

Figure. 1.

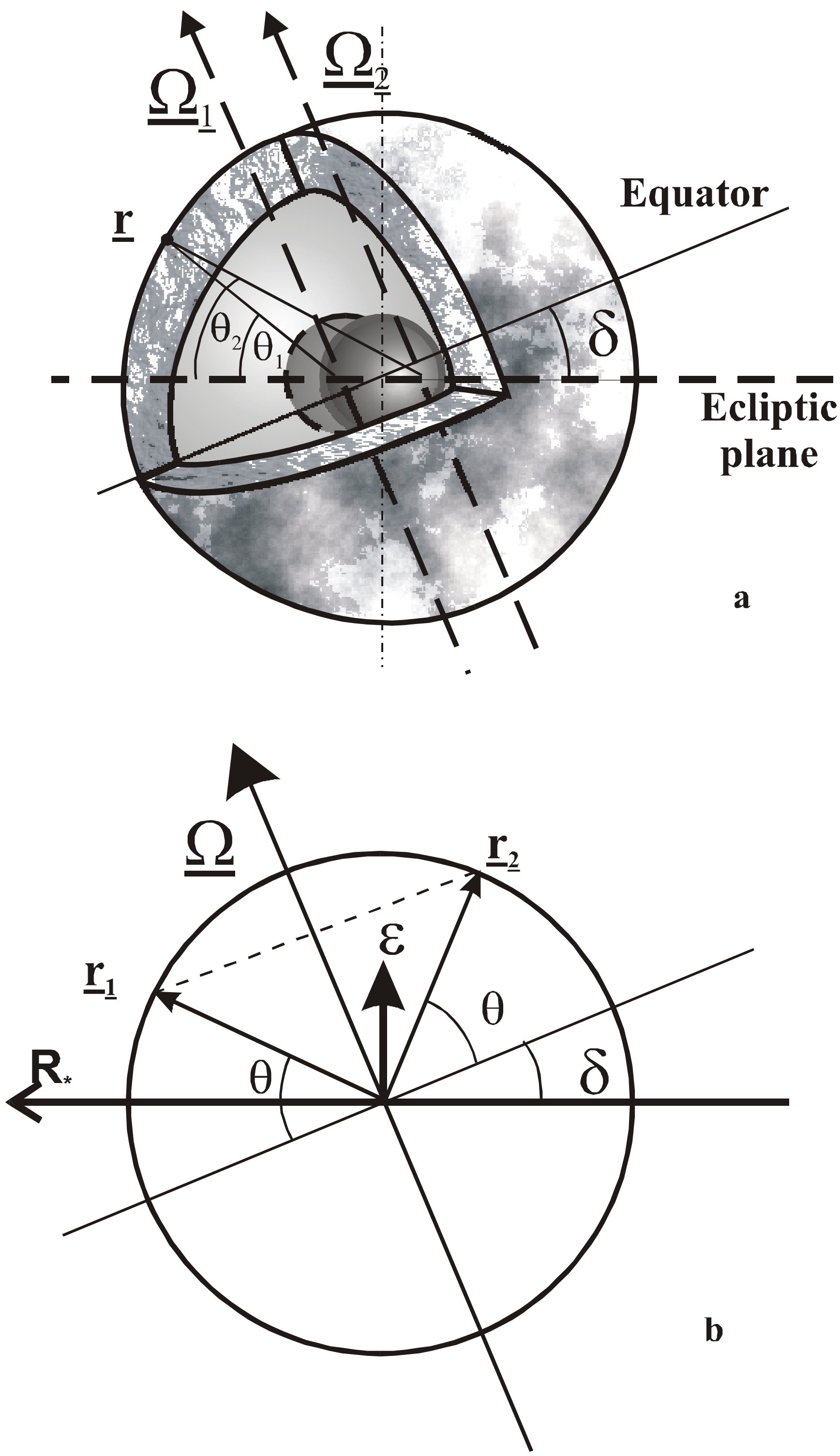

Figure. 2.

The above conclusions are qualitative. The model deals with quasi-stationary lithosphere and only accounts for density variations with depth. It is obvious that geological structural heterogeneity have to influence significantly the effects from the astronomical factors.

Let us further note that the dependence of free energy variations on the depth of the ellipsoidal layer, over which the density is integrated, is quadratic. It is interesting that these variations do not depend on model assumptions about the nature of density variations with depth. They depend only on the small depth of the layer relative to the Earth radius; the first terms of the respective ratio provide the main contribution in integral (14). The term, which is linear with depth, vanishes because the multiplier of this term, which is proportional to the pressure at zero depth, is zero. Instead, this depends on the choice of approximation of free energy in the linear elasticity theory. It is justified by the fact that accounting for non-linear terms (e.g. a cubic in free energy stress) is supposed to decrease the speed of sound with increased pressure, in contradiction with the actual observations in the Earth crust..

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