ASCENDING CURRENTS CAUSED BY BOTTOM OSCILLATIONS

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ABSTRACT

The ascending current formed in fluid on bottom area oscillations is investigated. The mathematical model motivating the cause of the phenomenon is created. The experiments carried out in water and glycerin allowed to reveal the fluid viscosity role in current forming. The finding indicates the possibility of sudden vertical exchange increase in ocean above underwater earthquake source.

INTRODUCTION

Tsunami waves are the most well-known seismic movement effects impacting on the ocean. Hydrodynamics phenomena localized near an earthquake epicenter are out-of-the-way and practically unstudied. The huge energy of tsunami waves in the far-field indicates the water layer in the tsunami source has catastrophic disturbances.

The descriptions of such disturbances obtained from eyewitnesses are included in the tsunami catalogs [Soloviev, Go, 1974, 1975; Soloviev at all, 1992, 2000]. There is a very intensive water movement due to an earthquake in the description: "…ocean looked like a rough boiler". It is reported about turbidity of water. The most effective event occurred in 1687. The ship drifting in the Pacific 600 km off the South America coast (the ocean depth was more than 4 km!) went through the dreadful seaquake, the usually green water seemed to become white. Having scooped up the water, they saw it was mixed with sand.

The changes of ocean surface temperature [Levin at all, 1998] and chlorophyll concentration [Levin at all, 2001] have been recently found. Strong underwater earthquakes followed these phenomena.

All these phenomena look to the possibility of a sharp vertical exchange increase as a result of the underwater earthquake. However, the vertical exchange intensification mechanism has been doubtful up to recent days.

We stated experimentally in the work [Nosov, Skachko, 2000] that the ascending current is formed above the oscillating bottom area under certain conditions. The current is able to increase significantly the vertical exchange. The aim of this work is the further experimental and theoretical studying of the currents induced by bottom oscillations. Aside from the main intent, this work is very important to understand the processes in tsunami source.

MATHEMATICAL MODEL

Mathematical model was created to find out theoretically the processes followed the bottom oscillations. It was built on the Euler equations:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}, \nabla)\vec{v} = -\frac{\nabla p}{\rho} + \vec{g}\,, \tag{1}$$

$$\mathrm{div}(\vec{v}) = 0\,. \tag{2}$$

The fluid current was assumed as a superposition of slow (average) and fast (oscillating) movements:

$$\vec{v} = \langle\vec{v}\rangle + \vec{v}', \quad p = \langle p\rangle + p', \tag{3}$$

Notice, that we treat the oscillating movement as a linear response to bottom oscillations. Substituting the formula (3) into the equations (1)-(2) and taking an oscillating period average we get the equations for the slow motion, where we neglect the nonlinear term $\left(\langle\vec{v}\rangle, \nabla\right)\langle\vec{v}\rangle$

$$\begin{aligned} &\frac{\partial\langle\vec{v}\rangle}{\partial t} = -\frac{\vec{\nabla}\langle p\rangle}{\langle\rho\rangle} + \vec{g} - \left\langle\left(\vec{v}', \vec{\nabla}\right)\vec{v}'\right\rangle, \\ &\mathrm{div}\langle\vec{v}\rangle = 0. \end{aligned} \tag{4}$$

The equations (4) differ from the common Euler equations with a new term

$$\vec{f} = -\left\langle\left(\vec{v}', \vec{\nabla}\right)\vec{v}'\right\rangle, \tag{5}$$

which can be interpreted as an external mass force.

An auxiliary problem was solved to calculate the force components. It was done in terms of fluid velocity potential $F(r, z, t)$

$$r^{-1}\left(r F_r\right)_r + F_{zz} = 0, \tag{6}$$

$$F_{tt} = -g F_z, \quad z = 0, \tag{7}$$

$$F_z = \eta_t, \quad z = -H. \tag{8}$$

The bottom movement was assumed as:

$$\eta(r, t) = \eta_0 \exp(-r^2 a^{-2})\,\theta(t)\sin(\omega t). \tag{9}$$

The solution of the problem (6)-(8), which is obtained using Laplace and Fourier-Bessel transforms, is:

$$\begin{aligned} F(r, z, t) = \eta_0\omega \int_0^\infty &\frac{J_0(kr)\,\mathrm{ch}(kz)\,X(k)}{\mathrm{ch}(kH)} \times \\ &\times \frac{\left[gk\,\mathrm{th}(kH)\,\mathrm{th}(kz) + gk\right]\cos(t\,(gk\,\mathrm{th}(kH))^{1/2}) - \left[\omega^2\,\mathrm{th}(kz) + gk\right]\cos(\omega t)}{gk\,\mathrm{th}(kH) - \omega^2}\,dk. \end{aligned} \tag{10}$$

The velocity components u' and w' were calculated using the fluid velocity potential: $u' = F_r$, $w' = F_z$.

The time and spatial components of the functions u' and w' are disintegrated under the condition $\nu > g^{1/2}H^{-1/2}$ (it is in accordance with the conditions of the experiment describing below) and the following expression is valid:

$$\{u'(r, z, t), w'(r, z, t)\} = \{u'_0(r, z), w'_0(r, z)\}\cos(\omega t). \tag{11}$$

The radial R and the vertical Z components of the force $\vec{f}$ after taking the oscillating period average are:

$$R = -\frac{1}{2}\left[u_0' \frac{\partial u_0'}{\partial r} + w_0' \frac{\partial u_0'}{\partial z}\right], \tag{12}$$

$$Z = -\frac{1}{2}\left[u_0' \frac{\partial w_0'}{\partial r} + w_0' \frac{\partial w_0'}{\partial z}\right]. \tag{13}$$

Notice that the force $\vec{f}$ cannot generate an eddy. That is easy to show by applying the operation "rot" to the equation (4). Taking into account the well-known vector analysis formalism and the expression $\vec{v}' = grad\ F$, we obtain that:

$$\frac{\partial}{\partial t} \mathrm{rot}\langle \vec{v} \rangle = 0 .$$

Hence, the force $\vec{f}$ cannot generate an eddy. In order to describe it we are to consider the viscosity impact, which cannot be neglected in a thin near bottom layer.

There are two approaches to consider the viscosity impact. The first one, quite complicated is connected with the direct solving of the problem where the viscous fluid linear response to bottom movements is investigated. The second approach is intended to make a "correction" of the solution obtained before in the frameworks of potential theory in such a way, that the tangent fluid velocity component satisfies the condition $u\big|_{bottom} = 0$.

VISCOSITY ROLE

In order to consider the viscosity role in the present problem let us follow the approach described in the treatise [Landau, Lifshitz, 1986]. As it is shown in that study, the fluid movement induced by the oscillations of a body (we consider the bottom oscillations) is eddy one only in some layer in the neighborhood of this body. Then the fluid movement passes fast into potential one on distance increasing. Notice that we consider the linear response.

The potential theory equations turned out partly valid in the neighborhood of the body surface. According to the potential theory, the current velocity component that is normal to the surface fulfills the necessary boundary condition $w\big|_{bottom} = 0$. Therefore, the true behavior of this component will not diverge from the potential theory result, if we take into account the viscosity influence.

But the true behavior of the tangent component will sufficiently differ from the solution obtained in frameworks of the potential theory. The tangent component will vary noticeably, whereas its value is to coincide with the tangent bottom movement velocity. In case of harmonic oscillations it is easy to state the nature of the difference.

If an infinity flat surface oscillates harmonically in its plane (Oxy) as

$$u = u_0 e^{-i\omega t}, \tag{14}$$

the motion of viscous fluid osculating to the surface is given as (according to the analytical solution by [Landau, Lifshitz, 1986]

$$v = u_0 e^{-\frac{z^*}{\delta}} e^{i\left(\frac{z^*}{\delta} - \omega t\right)}, \tag{15}$$

where z * is a distance from the oscillating surface. It implies that there are transverse waves in a viscous fluid. Such waves are damped on the increasing of the distance from oscillating surface generating them. The value $\delta = \sqrt{2\nu\omega^{-1}}$ is called “the penetration depth of viscous waves”, where ν is a fluid viscosity, ω is an oscillate bottom frequency.

Using (15), we can correct the tangent velocity obtained before in the frameworks of potential theory in such a way, that it satisfies the condition $u|_{bottom} = 0$. The correction to the expressions (11) for the velocity components gives:

$$u'(r,z,t) = u'_0(r,z)\left[\cos(\omega t) - e^{-\frac{z^*}{\delta}}\cos(\omega t - \frac{z^*}{\delta})\right],$$
$$w'(r,z,t) = w'_0(r,z)\cos(\omega t), \tag{16}$$

where $z^* = z + H$ is the distance from the bottom. Using (16), the corrected formulas for the radial and vertical components of the force are written as

$$R^{corr} = -\frac{1}{2}u'_0\frac{\partial u'_0}{\partial r}\left\{1 + e^{-\frac{2z^*}{\delta}} - 2e^{-\frac{z^*}{\delta}}\cos(\frac{z^*}{\delta})\right\} -$$
$$-\frac{1}{2}w'_0\left[\frac{\partial u'_0}{\partial z}\left\{1 - e^{-\frac{z^*}{\delta}}\cos(\frac{z^*}{\delta})\right\} + \frac{u'_0}{\delta}e^{-\frac{z^*}{\delta}}\left\{\cos(\frac{z^*}{\delta}) - \sin(\frac{z^*}{\delta})\right\}\right], \tag{17}$$

$$Z^{corr} = -\frac{1}{2}u'_0\frac{\partial w'_0}{\partial r}\left\{1 - e^{-\frac{z^*}{\delta}}\cos(\frac{z^*}{\delta})\right\} - \frac{1}{2}w'_0\frac{\partial w'_0}{\partial z}. \tag{18}$$

The term $-u'_0 w'_0/2\delta$ dominates in the radial force component on $z^* \to 0$. The values u'_0, w'_0 and δ are positive, hence, the radial force component is always negative near the bottom. Notice that, if we don’t consider the viscosity, the radial force component is positive, i.e. it is co-directed to the trend of the bottom oscillation amplitude decreasing. It is connected with the negativity of the derivatives $\partial/\partial r$ and $\partial/\partial z$ with respect to the functions u'_0 and w'_0.

So the negative radial force component trend due to the fluid viscosity provides the fluid undercurrent to the center. Therefore, the viscosity determines the eddy observed in experiments described below.

The viscosity impact on the force field forming nonlinear current is illustrated on Fig.1. It appears mainly in the immediate vicinity of oscillating bottom where the field direction reverses.

On calculating the force field shown on Fig. 1a the formulas (12), (13) are used, which coincided with an ideal (non-viscous) fluid. Such force field does not generate rotating flow. The force field shown on Fig.1b is calculated using (17), (18). In this case the viscosity impact was taken into account and the possibility of rotating flow appearance is not challenged.

EXPERIMENTS AND THEIR RESULTS

The base of experiment set-up was the rectangular basin with inner sizes of 220×220×220 mm. The transparent basin walls were made from plexiglas, thickness of 30 mm. There was a round opening in the basin bottom center. A piston was placed in the opening. The piston oscillated harmonically in vertical direction. The amplitude and frequency of oscillations can

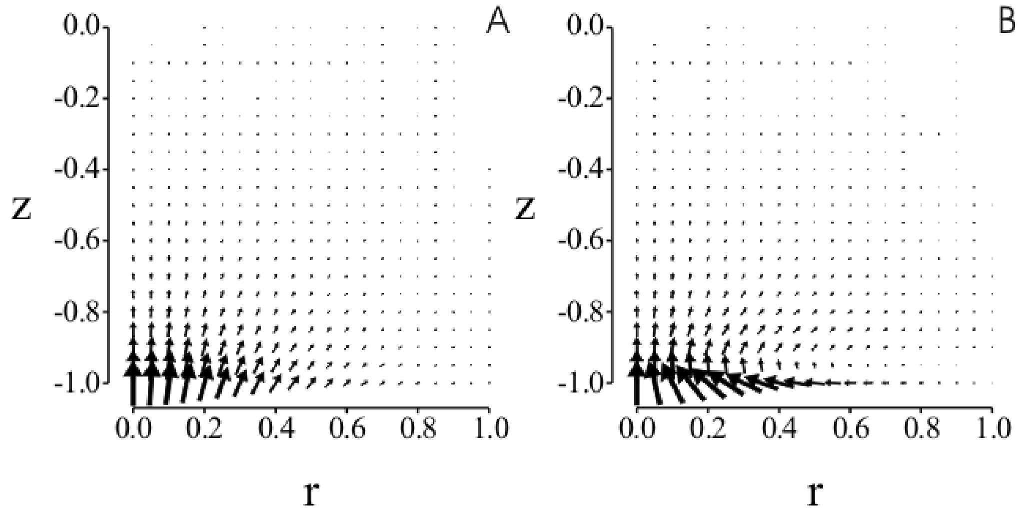

Fig. 1. Spatial force distribution obtained not taking into account (A) and taking into account (B) the viscosity. Computation was made for the bottom movement law (18) on a=0.5; δ=0.1.

be settled. The piston moved the elastic central bottom area. The spatial amplitude distribution of bottom oscillations approximately accorded with the exponential law:

$$\eta(r) = \eta_0 \exp(-r^2 a^{-2}) \ ,$$

where r is the distance from the central bottom area, a is a character area size; in case of the experiment a~20.

The experiment frequent and amplitude ranges were 5 – 35 Hz and 0.6 – 4.5 mm respectively. The piston was moved by a spring-eccentric mechanism. There were two series of experiments where basin was filled up with: 1) water; 2) glycerin. The fluid depth was 100 mm in both series. In order to visualize the dynamic processes inside the “light knife” technique was used. The light flat was perpendicular to the bottom surface and crossed the movable area center. We used chalk particles in water and aluminum particles in glycerin as tracers. The chalk suspension was injected into space near the bottom directly above the piston. The aluminum particles were distributed evenly in a volume. The sedimentation

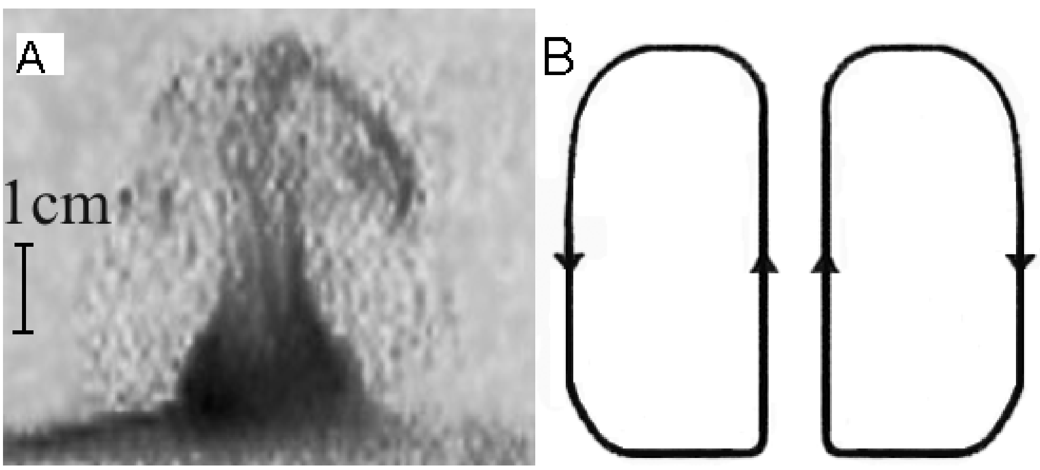

Fig. 2. Ascending current which is formed in water on bottom oscillations (A). Current scheme (B).

velocity was far less than any of the investigated current velocities. The dynamic processes inside were registered by a digital video camera.

Fig. 2a illustrates the appearance of vertical current directly above the piston center and the process of a slow fluid settle on the periphery. The order of velocity magnitude was about 5 mm/s. The process scheme is shown on Fig. 2b.

The experiments with glycerin facilitate the observation of current "fine structure" in a viscous layer near the bottom. The fact is that the penetration depth of viscous waves is defined by the formula $\delta = \sqrt{2\nu\omega^{-1}}$, water and glycerin viscosities have values of 0.01 cm^2/s and 6.8 cm^2/s respectively. Hereby, on the oscillating frequency of 10 Hz the viscous layer thickness is 0.17 mm and 4.7 mm for water and glycerin respectively. I.e. the value of δ for glycerin exceeds that for water almost by 30 times, which facilitates observing of processes in viscous layer.

Fig. 3 shows a character current picture obtained during glycerin experiments. This photo was obtained under the bottom oscillation frequency of 15 Hz and the oscillation amplitude of 4.5 mm. The current velocity reaches the value of 5 mm/s in the middle part of the current. As it can be seen from the figure, the current forms sui generis circulating contours near the

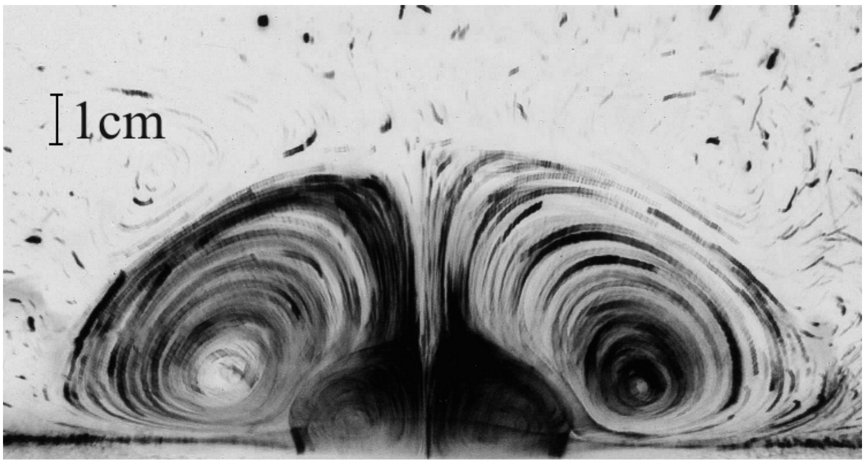

Fig. 3 (A). Ascending current, which is formed in glycerin on bottom oscillations.

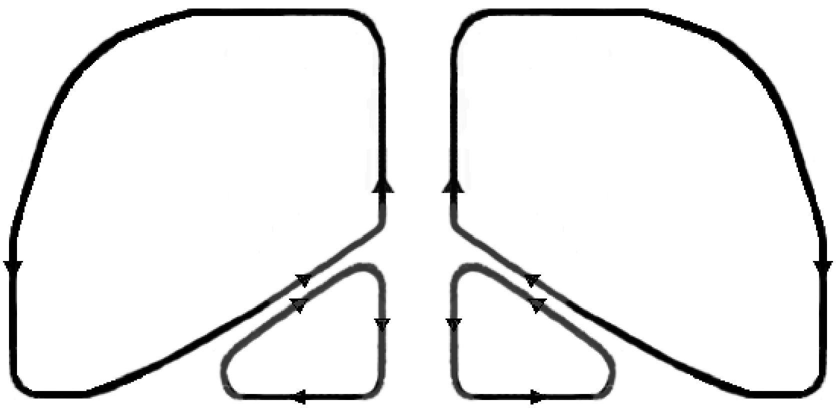

Fig. 3 (B). Current scheme.

bottom. In the neighborhood of the bottom the fluid current directs from the center at the periphery, then trajectory encloses making up a toroidal eddy. The toe of this eddy is situated in the viscous layer above the movable bottom area. The eddy causes moving of fluid in upper layers. It leads to generating of a bigger upper toroidal eddy. It is important to notice the upper eddy rotates in such a way that the fluid moves upward above the movable area center. It is very interesting that an analogous fluid current was obtained in the work [Ostrovsky, Papilova, 1979] under acoustical currents studying.

Thus, we can conclude the following. The possibility of ascending current formation above the oscillating bottom is shown in the experiments carried out in this work. It is defined that the current nature is nonlinear and its cause is a fluid viscosity. It is interesting to notice the current can penetrate a fluid layer at the distances in few orders more than the penetration depth of viscous waves. The analytical approaches used in this work motivate current forming and provide methods to create a mathematical model.

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