AMPLITUDE EVOLUTION AND RUNUP OF SOLITARY WAVES ON A SLOPING PLANE

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ABSTRACT

The runup of long waves on the sloping planes is described by the analytical solutions of the long wave equations with special initial conditions, proper approximations and boundary conditions. These studies are also verified by experimental data. It is convenient to test the numerical methods by comparing with analytical results. In this paper, the propagation and coastal amplification of solitary wave on a sloping plane is investigated numerically. The computed shape and amplitude evolution on the plane slope are compared with the existing analytical and experimental results. The performance of the numerical method is also discussed.

1. INTRODUCTION

The motions of long waves at shallower depths near the shoreline, run-up and the following inundation have been studied using theoretical, experimental and numerical approaches. Various analytical solutions for runup of nonlinear waves on plane slopes have been given by Shuto (1967); Gjevik & Pedersen [1981]; Pedersen and Gjevik, [1983]; Kim *et. al*., [1983]; Synolakis [1987]; Pelinovsky and Mazova [1992]; Synolakis and Skjelbreia, [1993]; Pelinovsky *et al*., [1996]; Kanoglu, [1996]; Kanoglu and Synolakis, [1997]; Lin *et. al*., [1999]; Carrier and Yeh, [2002]. In analytical approaches the runup problem is studied either by using empirical formulae or by solving the governing equations for specific initial and boundary conditions. Experimental data on runup of solitary waves are given among others by Hall and Watts, [1953], Pedersen and Gjevik, [1983] and Synolakis [1987], Shankar and Jayaratne, [2002], Lee and Raichlen, [2002].

A detailed analytical and experimental study on the runup and amplitude evolution of solitary waves on plane beaches is given in Synolakis, [1987]. An exact solution to an approximate theory for non-breaking solitary waves was introduced to derive the maximum runup asymptotically. Laboratory experiments had been performed to support the theory and the satisfactory prediction of the climb of the wave on the slope and maximum runup by linear theory has been determined. Pelinovsky and Mazova [1992] investigated tsunami runup on a beach with two different parameters; the angle of bottom slope and the breaking parameter.

Titov, Synolakis, [1995a, 1995b, 1998], Imamura [1995], Yalciner *et al*., [2001[, Hubbard, Dodd, [2002[, Lynett *et. al*., [2002[, Lee and Raichlen, [2002[, Maiti and Sen, [1999[are some references of numerical studies on long wave runup. The computer program, TUNAMI-N2, used for the simulation of the propagation of long waves is developed by Prof. Imamura in Disaster Control Research Center in Tohoku University, Japan. TUNAMI-N2 is one of the key tools for developing studies for propagation and coastal amplification of tsunamis in relation to different initial conditions. It solves the nonlinear form of long-wave equations and depth averaged velocities with bottom friction by finite difference technique for the basins of irregular shape and bathymetry and provides us a very convenient tool to simulate tsunamis. Shuto, Goto and Imamura (1990), Goto and Ogawa, [1992], Imamura, (1995), Goto *et. al.* [1997], Yalciner *et. al.*, [2001], Yalciner *et. al.*, [2002] are some of the studies used TUNAMI-N2.

In this study particularly, the behavior of solitary wave on a sloping beach and the runup phenomenon by the non-linear numerical modeling (TUNAMI-N2) is studied. The shape of the solitary wave on the plane slope, the maximum positive amplitudes near the coastline are computed and compared with the analytical and experimental results [Demirbas, 2002].

2. NUMERICAL APPLICATION WITH SOLITARY WAVE

A solitary wave centered at a location x = X_1 when t = 0 has the following surface profile:

$$\eta(x,0) = \frac{H}{d}\sec h^2 \gamma(x - X_1) \tag{1}$$

$$\gamma = (\frac{3H}{4d})^{1/2} \tag{2}$$

Where H is the amplitude of solitary wave, d is the water depth at the toe of the sloping plane, X_1 is the distance from the specified location.

The linearized long wave equations for the canonical problem are solved by Synolakis, (1987) and the runup law is derived for the non-breaking solitary waves.

$$\gamma = (\frac{3H}{4d})^{1/2} \tag{3}$$

where R is the runup of solitary waves, β is the angle of sloping plane with horizontal.

The breaking condition of solitary waves on a sloping plane is presented by Gjevik & Pedersen [1981]:

$$\frac{\mathrm{H}}{\mathrm{d}} > 0.479(\cot\beta)^{\frac{-10}{9}} \tag{4}$$

This criterion has been reported to be in excellent agreement with laboratory data for solitary waves by Synolakis, [1987].

The canonical problem named by Tadepalli and Synolakis [1994], in wave runup is the determination of the runup of a long wave propagating over a constant depth region and then climbing up a sloping beach of constant slope. There are a few numbers of studies for different wave profiles on the canonical problem. We selected the canonical problem with a regular shaped basin of 10 km length and width. The water depth of the horizontal bottom is chosen as 30 m. On one side of the basin the plane beach is located with bottom slope of 1/20. The other boundaries are selected as open boundaries. The cross section of the basin along x direction is shown in Fig. 1. The initial wave is inputted near the center of the basin where the wave crest is parallel to the shoreline (along z axis) and thus the wave propagation is forced along x direction towards shore without dispersion.

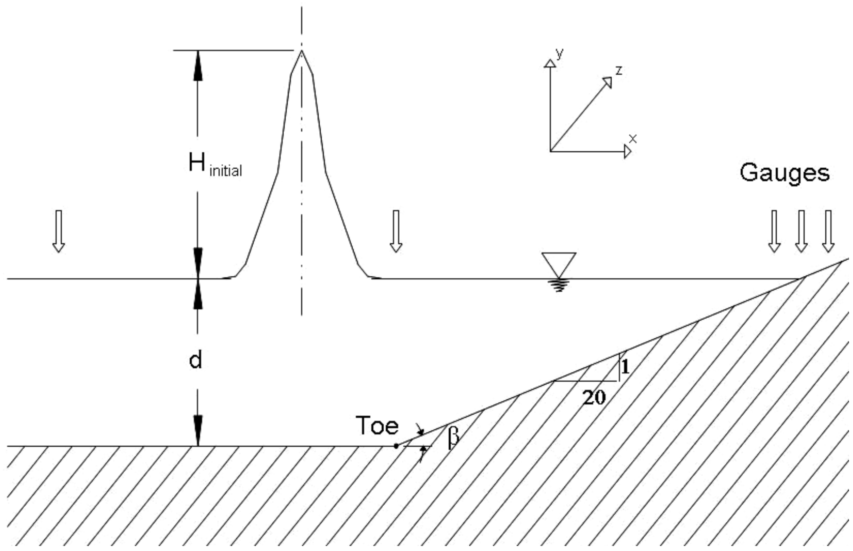

Figure 1. Cross Section of the Basin, Location of the Initial Solitary Wave and the Gauge Locations where the Water Surface Elevations are Computed.

The location of the calculated maximum water elevation near the shoreline obtained by the numerical model with finite difference method does not coincide with the location of the actual runup. In the numerical model, the elevation of the water is computed at the fixed locations of grid points. Obviously the smaller grid sizes result nearer maximum elevations to actual runup. In this application the smallest possible grid size is selected to obtain optimum run time and to obtain best possible comparison between experimental/ analytical and numerical results. The grid size and time step are selected as 20 m and 0.25 seconds respectively in order to satisfy stability in computation. The time histories of water surface elevations at different locations, the sea state at different time steps, the snapshots of the surface profile along the axis of wave propagation at specified time step, the maximum water elevation reached at every grid point throughout the domain during the simulation are computed and stored. By using the stored data the shape of the wave at different locations on the slope and the water surface along the axis of wave direction at specified time steps are presented in the following. The results are compared with the analytical and experimental data of Synolakis [1987].

There are two cases presented in Synolakis [1987]. They are also selected in this application. In these cases, the normalized height of incoming wave, (H/d) is 0.019 (non-breaking) and 0.040 (breaking).

The normalized water surface elevation (η) representing the climb of solitary wave at the toe of the slope, and at the shoreline for the non-breaking case on the 1:19.85 slope are shown in Fig. 2 as function of the dimensionless time. As seen from this Figure that the numerical model computes fairly consistent water surface fluctuation with experimental and analytical data.

The comparison is extended to check the water surface profile along the axis of propagation at different dimensionless time steps. The water surface profiles at different dimensionless time steps are given in Fig. 3 and 4. As seen from these figures that the numerical model provides fully consistent shape of the wave and amplitude evolution on the plane slope with the analytical and experimental results, especially on the wet part of the slope for the both breaking and non-breaking conditions of incoming solitary wave.

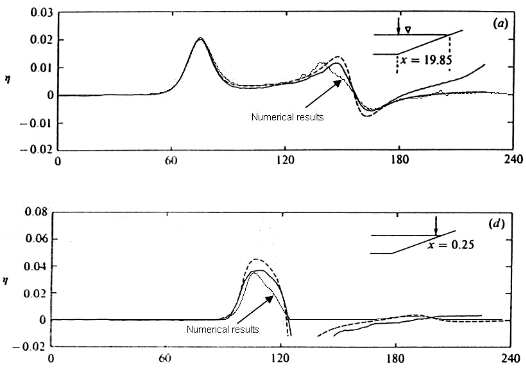

Figure 2. The normalized water surface elevation representing the climb of solitary wave at x = 19.85 (at the toe of the slope), and at x = 0.25 (at the shoreline) with H/d = 0.019 up a 1:19.85 slope as function of the dimensionless time. (—— experimental [Synolakis, 1987]; -----, analytical [Synolakis, 1987]; ——, numerical – this study).

The numerical experiments are repeated by using different incoming solitary waves. The normalized maximum positive wave amplitudes near the shoreline are computed for each experiment. The comparison of numerical data with the runup law and experimental data of Synolakis [1987] is given in Fig. 5. This figure shows that the distribution of the data points of numerical results show similar trend with the analytical and experimental data, but the numerical results stay below the others. The underestimation of maximum amplitude in numerical results comes from fixed grid size of the numerical solution.

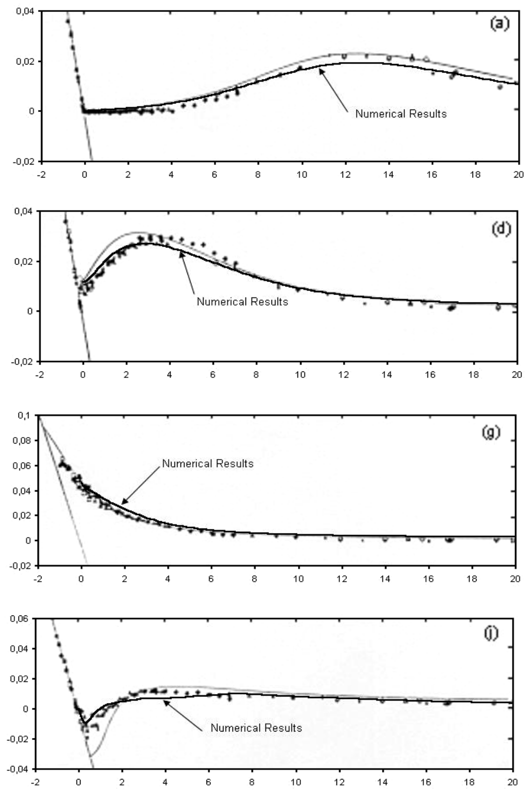

Figure 3. The normalized water surface profile representing the climb of solitary wave along the wave direction with H/d = 0.019 up a 1:19.85 slope as function of the normalized distance at different dimensionless time steps, (a) t = 25, (d) t = 40, (g) t = 55, (i) t = 65. (**....**, experimental [Synolakis, 1987]; ____-, analytical [Synolakis, 1987]; ——, numerical – this study).

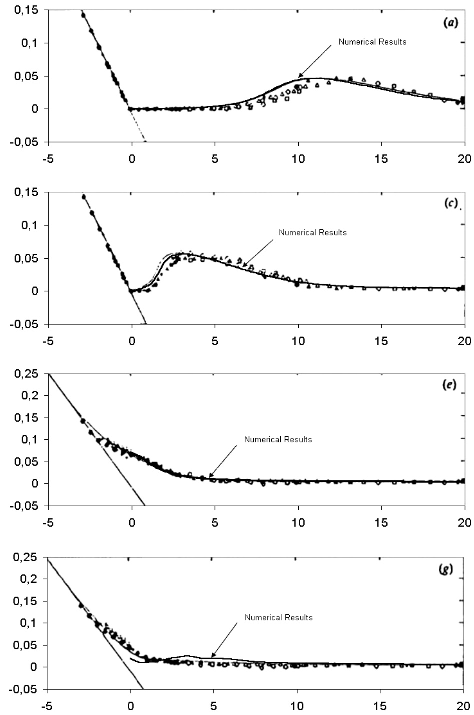

Figure 4. The normalized water surface profile representing the climb of solitary wave along the wave direction with H/d = 0.040 up a 1:19.85 slope as function of the normalized distance at different dimensionless time steps, (a) t=20, (c) t=32, (e) t=44, (g) t=56. [**....**, experimental [Synolakis, 1987]; _____-, analytical [Synolakis, 1987]; —— , numerical – this study).

3. Discussions of Results

The analytical results do not cover the non-linear terms of the long wave equations. Lin *et al.*, [1999] states that the numerical results of the depth averaged equations models predict smaller value of runup tongue. However analytical approach is consistent with the experimental results. Therefore it is shown that the numerical approach computes satisfactory agreement of water motion when the wave climbs on the slope. But the computation gives smaller runup on the slope at land.

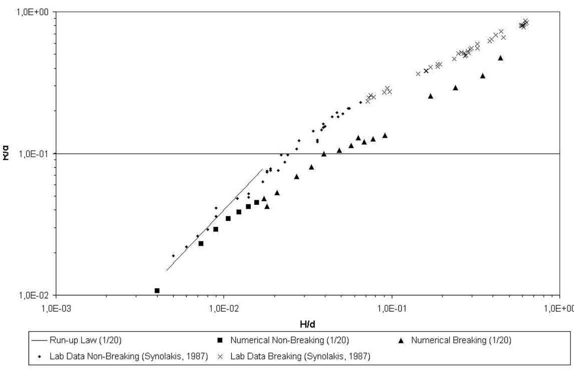

Figure 5. The Comparison of Numerically Computed Maximum Positive Wave Amplitudes near the Shoreline with the Runup law and Experimental Data Given in Synolakis, [1987]

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